Spectral Analysis and Heart Rate Variability: Principles and Biomedical Applications

Dr. Harvey N. Mayrovitz
Why Spectral Analysis?

Detection and characterization of cyclical or periodic processes present in physiological signals

Rhythms are present in nearly all physiological signals - but not always evident to the ‘naked eye’!
How do you extract spectral (frequency) components present in physiological signals?
Power Spectral Density

Amount of power per unit (density) of frequency (spectral) as a function of frequency

PSD describes how the power (or variance) of a time series is distributed with frequency!
Example with Simulated Signals
A+ B+ C
100 sec

OK Resolution

Dr. HN Mayrovitz
A+ B+ C
1000 sec

Much Better Resolution
Generating a time series signal from the Electrocardiogram
R- R Time Series

R - R Interval varies with time

\[(R - R)_i\]  \[\rightarrow\]  \[(R - R)_{i+1}\]

QRS interval
R-R Time Series

A. Supine rest

B. Mental Arithmetic

C. Exercise

D. Congestive Heart Failure
Heart Rate Variability
Heart Rate Variability (HRV)
Heart Rate Variability (HRV)

- Peripheral Vascular & Thermoregulatory
- Baroreceptors phase delay
- Sympathetic & Parasympathetic
- Respiratory Sinus Arrhythmia (RSA)
- Cardiac Vagal Activity Change

Frequency Bands:
- ULF: <0.003 Hz
- VLF: 0.003 - 0.04 Hz
- LF: 0.04 - 0.15 Hz
- HF: 0.15 - 0.40* Hz

Graphs showing RR interval, power spectrum, and frequency distribution.
RSA Main Source of HF peak
Respiratory Linkage to HF & LF

Parasympathetic (Vagus) Heart Rate Brake

Inspiration (inhibits vagus nerve outflow impulses)

- Centrally?
- Increased venous return

Baroreflex

Vagus ~ Fast ~ HF

Sympathetic ~ Slow ~ Phase Delay ~ LF

HR changes
Importance of Respiration

Peroneal nerve sympathetic

Paced Breathing At 0.2 Hz

Quiet Breathing

Apnea (100 sec)
Slow rate allows fuller expression of Ach effects
Resulting in greater HF power at lower frequencies

Note HR itself DOES NOT CHANGE!
Relationship to Neural Signals

R-R Interval

Cardiac Nerve Traffic

Sympathetic

Vagal
Enhancement of Sympathetic Modulation
24 Hour Recording

Physiological Correlates not known yet constitutes Largest Power!
Time Analysis of HRV uses standard deviation or variance of (normal) R-R intervals

Coefficient of variance = \( \frac{SD}{\text{mean}} \)

= \( \frac{SD_{NN}}{\text{mean}} \)
Spectral Analysis Considerations
For a given sampling rate the length of time a signal is sampled sets the Frequency Resolution.
Signal
80 cycles of a 1 Hz sine wave

Fourier Power Spectrum
Power Spectral Density (PSD)

Dr. HN Mayrovitz
Signal
40 cycles of a 1 Hz sine wave
Signal
20 cycles of a 1 Hz sine wave

Dr. HN Mayrovitz
Signal
10 cycles of a 1 Hz sine wave
Signal
5 cycles of a 1 Hz sine wave
Signal
2 cycles of a 1 Hz sine wave
Seperating frequency components requires adequate resolution
1.0 Hz

0.3 Hz

A + B
A
1.0 Hz

B
0.3 Hz

C
0.065 Hz

A + B + C
A+ B+ C
100 sec

OK Resolution

Dr. HN Mayrovitz
1000 sec

Much Better Resolution

Dr. HN Mayrovitz
De-Trending
R-R Interval series as obtained

Detrended Series

Power Spectral Density (PSD) of R-R Series

- Original
- Detrended

FFT

AR
Effect of Detrending

R-R Interval series as obtained

Detrended Series
Basic Definitions
Coherence Function - Degree of linear correlation as fn of frequency

\[ G_{xx}, G_{yy} \text{ and } G_{xy} \text{ are spectra of } x(t), y(t) \text{ and crosspectrum of } x \text{ and } y \]

\[ [K(f)]^2 \]
If \( x(k) \) is the \( k \)-th value of a time series of \( N \) samples with sampling period \( \Delta t \), its energy \( E \) is defined as:

\[
E = \sum_{k=0}^{N-1} \left| x(k) \right|^2 \Delta t
\]

The power \( P \) is then defined as:

\[
P = \frac{E}{N\Delta t} = \frac{1}{N} \sum_{k=0}^{N-1} \left| x(k) \right|^2
\]

For zero-mean time series, the power is equal to the variance of the sample of the \( N \) values \( x(k) \).
Aliasing Artifacts

When the sampling rate is greater than the Nyquist rate ($f_N$), the spectrum remains true. When the sampling rate is less than the Nyquist rate, the spectrum is aliased, and erroneous folded spectra are observed in the frequency domain.

- Sampling rate > $f_N$: True spectrum
- Sampling rate < $f_N$: Aliased spectrum
Windowing

- No windowing
- 10% cosine-taper
- Hann

Equation: $w(t) = \begin{cases} 1 & 0 \leq t \leq 100 \\ 0 & \text{otherwise} \end{cases}$
Autocorrelation Function

Measure of the dependence of time series values at one time on the values at another time.

Given the time series $x(n)$, $n=1, 2, \ldots N$, the autocorrelation function at lag $k$ is defined as:

$$R_{xx}(k) = \frac{1}{N-k} \sum_{n=1}^{N-k} x(n)x(n+k)$$

The value of the autocorrelation function at lag 0 is the power of $x(n)$, or its variance if the mean value of $x(n)$ is zero:

$$R_{xx}(0) = \frac{1}{N-k} \sum_{n=1}^{N-k} x(n)^2$$

Moreover, $\sqrt{R_{xx}(\infty)}$ is the mean value for random processes.
Left: unsmoothed FFT spectrum of blood pressure from a 8-h recording: this spectrum is characterized by a very high frequency resolution, but also by a very high estimation variance. Centre: the same spectrum smoothed by a moving average filter of order 250 (i.e., average over 250 adjacent spectral lines). Estimation variance is largely reduced, but the frequency resolution dramatically worsens and important spectral details may be lost at the lower frequencies. Right: broad-band spectrum obtained from the raw FFT spectrum by averaging adjacent spectral lines: in this case the number of lines to average increases with the frequency from 1 to 250. The desired reduction of the estimation variance is obtained at the highest frequencies preserving the original frequency resolution at the lowest frequencies.
Sensitivity to initial conditions. Small changes in initial conditions lead to totally different behaviour patterns after a certain time (here 14 cycles). This sensitivity to initial conditions may be quantified by means of the largest Lyapunov exponent.
Another Type of Experiment

Dr. H. N. Mayrovitz
Experiment

Blood Flow Finger 2
Blood Flow Finger 4
PPG
RESP

60 sec/div
45 minutes

Dr. HN Mayrovitz
Experiment

Blood Flow Finger 2

Blood Flow Finger 4

PPG

RESP

45 seconds

Dr. HN Mayrovitz
Why 2 peaks?

45 minute sample
Physiological signals whose spectral content changes with time

Principle of STFT
Short Time Fourier Transform

Dr. H. N. Mayrovitz
PPG - 45 minute sample using STFT

Dr. HN Mayrovitz
Principles of Short Time Fourier Transform Analysis

Seg
1
T1

2
S
T2

3

4

5

6

7

8

9

10
T10

T1 = T_{total} - \frac{N_{\text{precision}}}{F_s} = 1200 - 819.2 = 380.7 \text{ sec}

\frac{(N_{\text{segs}} - 1) \times S}{20} = 9 \times 42.3 \text{ sec} = 380.7 \text{ sec}

T_{\text{total}} = 20 \text{ minutes} = 1200 \text{ sec}, \ F_s = 20 \text{ s/sec}

N_{\text{precision}} = 16384 = \frac{16384}{20} = 819.2 \text{ sec}

F_{\text{precision}} = \frac{1}{819.2} = 0.0012 \text{ Hz}

Dr. HN Mayrovitz
RESP
45" sample using STFT
Flow F4 45”
HNM: 20” moor signal at 20 s/sec = 24000 pts on left hand =24000/20=1200 sec
precision=16384, #seg=10 therefore step =1756
precision ~ 16384/20 = 819.2 sec; step ~ 846/20 = 42.3 sec

Precision=number of points per spectrum
Step = S = number of points from start of one spectrum to start of the next

Dr. HN Mayrovitz
HNM: 20” moor signal at 20 s/s = 24000 pts on LH, both with precision = 8192, Fs = 20

A
#seg=70
step=229

B
#seg=10
step=1756
<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Description</th>
<th>Frequency Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis of Short-term Recordings (5 min)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-min total power</td>
<td>ms²</td>
<td>The variance of NN intervals over the temporal segment</td>
<td>≈≤0.4 Hz</td>
</tr>
<tr>
<td>VLF</td>
<td>ms²</td>
<td>Power in VLF range</td>
<td>≤0.04 Hz</td>
</tr>
<tr>
<td>LF</td>
<td>ms²</td>
<td>Power in LF range</td>
<td>0.04-0.15 Hz</td>
</tr>
<tr>
<td>LF norm</td>
<td>μu</td>
<td>LF power in normalized units LF/(total power-VLF)x100</td>
<td></td>
</tr>
<tr>
<td>HF</td>
<td>ms²</td>
<td>Power in HF range</td>
<td>0.15-0.4 Hz</td>
</tr>
<tr>
<td>HF norm</td>
<td>μu</td>
<td>HF power in normalized units HF/(total power-VLF)x100</td>
<td></td>
</tr>
<tr>
<td>LF/HF</td>
<td></td>
<td>Ratio LF [ms²]/HF[ms²]</td>
<td></td>
</tr>
<tr>
<td>Analysis of Entire 24 Hours</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total power</td>
<td>ms²</td>
<td>Variance of all NN intervals</td>
<td>≈≤0.4 Hz</td>
</tr>
<tr>
<td>ULF</td>
<td>ms²</td>
<td>Power in the ULF range</td>
<td>≤0.0003 Hz</td>
</tr>
<tr>
<td>VLF</td>
<td>ms²</td>
<td>Power in the VLF range</td>
<td>0.003-0.04 Hz</td>
</tr>
<tr>
<td>LF</td>
<td>ms²</td>
<td>Power in the LF range</td>
<td>0.04-0.15 Hz</td>
</tr>
<tr>
<td>HF</td>
<td>ms²</td>
<td>Power in the HF range</td>
<td>0.15-0.4 Hz</td>
</tr>
<tr>
<td>α</td>
<td></td>
<td>Slope of the linear interpolation of the spectrum in a log-log scale</td>
<td>≈≤0.04 Hz</td>
</tr>
<tr>
<td>Variable</td>
<td>Units</td>
<td>Normal Values (mean±SD)</td>
<td></td>
</tr>
<tr>
<td>-------------------------------------------------------</td>
<td>-------</td>
<td>-------------------------</td>
<td></td>
</tr>
<tr>
<td>Time Domain Analysis of Nominal 24 hours</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SDNN</td>
<td>ms</td>
<td>141±39</td>
<td></td>
</tr>
<tr>
<td>SDANN</td>
<td>ms</td>
<td>127±35</td>
<td></td>
</tr>
<tr>
<td>RMSSD</td>
<td>ms</td>
<td>27±12</td>
<td></td>
</tr>
<tr>
<td>HRV triangular index</td>
<td></td>
<td>37±15</td>
<td></td>
</tr>
<tr>
<td>Spectral Analysis of Stationary Supine 5-min Recording</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total power</td>
<td>ms²</td>
<td>3466±1018</td>
<td></td>
</tr>
<tr>
<td>LF</td>
<td>ms²</td>
<td>1170±416</td>
<td></td>
</tr>
<tr>
<td>HF</td>
<td>ms²</td>
<td>975±203</td>
<td></td>
</tr>
<tr>
<td>LF</td>
<td>nu</td>
<td>54±4</td>
<td></td>
</tr>
<tr>
<td>HF</td>
<td>nu</td>
<td>29±3</td>
<td></td>
</tr>
<tr>
<td>LF/HF ratio</td>
<td></td>
<td>1.5-2.0</td>
<td></td>
</tr>
</tbody>
</table>